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CHANGES OF A POROUS STRUCTURE IN FLOW OF A MONODISPERSE MEDIUM

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The motion of fluids with suspended particles in porous media is considered. A mathematical model for the interaction of a monodisperse suspension with a porous structure is proposed. Changes in the parameters of the medium and the flow are studied for equilibrium regimes.

The need to study the motion of fluids containing dispersed phases in porous media arises in many practical problems. Among these problems are purification of fluids by filtration, reduction in the conductivity of the near-well zone of a bed due to penetration of a flushing fluid filtrate into it, evaluation of the effects of reinforcing of the emulsified phase by mechanical impurities in working wells, etc. [1–3].

Semiempirical models are widely used for analysis of these phenomena. The filtration of low-concentration suspensions was considered by Shekhtman [2]. He used approximate equations and assumed that the rate of entrainment of particles of the flow in the medium was proportional to the deviation of the concentration from a certain equilibrium value. At the same time, a comparison with the experiments of [4, 5] shows that, in many cases, the kinetics of sedimentation is strongly affected by the velocity of filtration. Schematization of a pore space used in [6, 7] introduces new additional empirical constants, and the system of balance equations obtained for averaged quantities is approximate [5]. A correct form of this system (without a continuity equation for liquid phase) is given in [8].

In the method of network modeling [1], a porous medium is replaced by a fixed set of points (pores) connected by channels with randomly distributed radii. Here the fundamental difficulties are related to the significant limitation of the network size, considerable number of additional parameters, and application of a number of percolation hypothesis that are substantially quasistationary in character.

The present paper extends the approach proposed in [9] to the motion of a monodisperse suspension (the corresponding approximate model is given in [10]). This makes it possible not only to formulate an exact system of equations for integral characteristics in a comparatively simple way but also to describe changes in the structural parameters of the medium in the sedimentation process. The class of equilibrium regimes obtained in this case serves as a basis for construction of fundamental relations for integral models.

1. Formulation of the Model. We consider the motion of a fluid carrying suspended particles of the same radius $\bar{\rho}$ through a porous medium. The flow is one-dimensional and is along the x axis. Let t be time, $v(x,t)$ be the velocity of filtration, $m(x,t)$ be the porosity, and $N(x,t)$ and $c(x,t)$ be the number and volume concentrations of particles in the flow. The solid and liquid phases in the suspension, and the matrix of the porous medium are considered incompressible.

A volume element of a porous medium at an arbitrary point x is assumed to consist of sieve-like layers that are adjacent to each other and perpendicular to the flow and whose thickness $l(x,t)$ varies in time and space. The pores in a layer are treated as right circular cylinders with an axis of length l along the flow and with randomly distributed radii r. Let $M_s(x,t)$ be the number of such channels per unit area of the layer,

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$\varphi_s(r, x, t)$ be the distribution of the channels in the layer by their radii. and $M(x, t)$ be the number of pores per unit volume. The layers are treated as probability-independent in the sense that after a particle passes through a certain channel of the layer, it comes to the entrance of a channel of radius r of the next layer with the probability determined only by the function $\varphi_s(r, x, t)$.

The function φ_s is nonnegative and satisfies the condition

$$\int_0^\infty \varphi_s \, dr = 1. \tag{1}$$

The porosity m is related to M, φ_s, and l by the equality

$$m = \pi l M \int_0^\infty r^2 \varphi_s \, dr. \tag{2}$$

There are $1/l$ layers per unit length along the flow, and, therefore, $M_s = lM$. Hence, the degree of perforation (surface porosity) and (volume) porosity coincide, so that this schematization of the pore space is statically isotropic [11].

Let us consider a single layer with cross-sectional area S. The number of particles that pass through its channels of radius r in time dt is equal to $vS\,dt\,Nr^2\varphi_s\,dr \Big/ \int_0^\infty r^2\varphi_s\,dr$. A particle is entrained in this channel if its radius is smaller than the radius of the particle. Therefore, over this period of time, $vS\,dt\,N \int_0^{\bar\rho} r^2\varphi_s\,dr \Big/ \int_0^\infty r^2\varphi_s\,dr$ particles are entrained in the layer. Since the volume Sdx contains dx/l layers, we have

$$\frac{\partial(mN)}{\partial t} + \frac{\partial(vN)}{\partial x} = -vN \int_0^{\bar\rho} r^2\varphi_s\,dr \Big/ \left(l \int_0^\infty r^2\varphi_s\,dr \right).$$

The relation between the volume concentration c and the number concentration N is $c = 4\pi\bar\rho^3 N/3$, and, therefore, the equation obtained can be written as

$$\frac{\partial(mc)}{\partial t} + \frac{\partial(vc)}{\partial x} = -vc\frac{q}{l}, \qquad q = \int_0^{\bar\rho} r^2\varphi_s\,dr \Big/ \int_0^\infty r^2\varphi_s\,dr. \tag{3}$$

The quantity q in (3) is the probability of the event that a particle that comes to the layer is entrained in it. This quantity depends on x and t, and is considerably affected by the particle size and the initial state of the porous structure. If, for example, at $t = 0$, pore channels are absent in the interval $0 < r < \bar\rho$, the source term in (3) is zero at all times, and only simple transfer of the suspension occurs.

The cross-sectional area of channels of radius r in the layer is $SM_s\pi r^2\varphi_s\,dr$. The volume of suspension passing through this area in time dt is $(v/m)\,dt\,SM_s\pi r^2\varphi_s\,dr$, and it carries $N(v/m)\,dt\,SM_s\pi r^2\varphi_s\,dr$ particles. Channels with radii $r > \bar\rho$ remain permeable and their radii are unchanged. Some of the channels with radii $r < \bar\rho$ become blocked up. The number of these in the volume Sdx is $(dx/l)N(v/m)\,dt\,SM_s\pi r^2\varphi_s\,dr$. Therefore, the number density $M\varphi_s$ of cylindrical pores of radius r must satisfy the equation

$$\frac{\partial(M\varphi_s)}{\partial t} = -\frac{3r^2\varphi_s vc}{4\pi\bar\rho^3 l} H(\bar\rho - r) \Big/ \int_0^\infty r^2\varphi_s\,dr. \tag{4}$$

From here on, $H(x)$ denotes the Heaviside step function: $H(x) = 1$ for $x > 0$ and $H(x) = 0$ for $x < 0$.

From (4), the normalization condition (1), and the definition of q in (3), we obtain the following expression for the rate of change in the number of pores in unit volume:

$$\frac{\partial M}{\partial t} = -vc\frac{3}{4\pi\bar{\rho}^3}\frac{q}{l}. \tag{5}$$

According to (3) and (5), the number of pores blocked up in unit time coincides with the number of particles entrained in the medium in unit time. As follows from (4), the number density of pores with radii of cylinders $r > \bar{\rho}$ does not change with time. The total number of pores is also conserved. At the same time, in view of (4) and (5), we have

$$M\frac{\partial \varphi_s}{\partial t} = \frac{3\varphi_s vc}{4\pi\bar{\rho}^3 l}\left(\int\limits_0^{\bar{\rho}} r^2\varphi_s\,dr - r^2H(\bar{\rho}-r)\right)\Big/\int\limits_0^{\infty} r^2\varphi_s\,dr.$$

Therefore, the contribution of pores with radii greater than the radius of particles increases with time. However, a similar tendency is also characteristic for pores with radii $0 < r < r_*$, where $r_* = r_*(x,t)$ is defined by the equality

$$r_*^2 = \int\limits_0^{\bar{\rho}} r^2\varphi_s\,dr.$$

The reason for such behavior is that, as compared to pores with radii $r < r_*$, the number of pores with radius $r_* < r < \bar{\rho}$ is smaller and the probability that they become blocked up is lower. It can be shown that the quantity r_* decreases with time.

Next, in view of (2),

$$\frac{\partial m}{\partial t} = l\frac{\partial}{\partial t}\left(M\pi\int\limits_0^{\infty} r^2\varphi_s\,dr\right) + M\pi\int\limits_0^{\infty} r^2\varphi_s\,dr\frac{\partial l}{\partial t},$$

and, therefore, as follows from (4),

$$\frac{\partial m}{\partial t} = \frac{m}{l}\frac{\partial l}{\partial t} - vc\frac{3}{4\bar{\rho}^3}\int\limits_0^{\bar{\rho}} r^4\varphi_s\,dr\Big/\int\limits_0^{\infty} r^2\varphi_s\,dr.$$

Since all phases involved in the process are incompressible, the volume of the particles entrained in the medium in unit time must coincide with the volume of all pores blocked up by these particles over the same period of time. From this fact and from (3), we obtain the following equation for porosity:

$$\frac{\partial m}{\partial t} = -vc\frac{q}{l}. \tag{6}$$

In addition, the thickness of the layers l varies in such a way that the following equation is satisfied:

$$m\frac{\partial l}{\partial t} = vc\left(\frac{3l}{4\bar{\rho}^3}\int\limits_0^{\bar{\rho}} r^4\varphi_s\,dr\Big/\int\limits_0^{\infty} r^2\varphi_s\,dr - q\right). \tag{7}$$

Because of incompressibility, the blocking of the pore space by solid particles does not remove the liquid phase from the flow, and, hence, [5]

$$\frac{\partial(m(1-c))}{\partial t} + \frac{\partial(v(1-c))}{\partial x} = 0.$$

From this and from (3) and (6), we obtain the following equation for the velocity of filtration of the suspension:

$$\frac{\partial v}{\partial x} = 0. \tag{8}$$

Taking the velocity of suspended particles equal to the flow velocity, we assume Darcy's law for the suspension as a whole [2]:

$$v = -\frac{k}{\mu}\frac{\partial p}{\partial x}. \tag{9}$$

Here p is the pressure in the suspension, μ is its viscosity, and k is the permeability.

To connect the permeability variations with the flow parameters, we fix the pressure differential $\partial p/\partial x$ at the point x. In each layer, the flux $(v/m)\, dt\, SM_s \pi r^2 \varphi_s\, dr$ comes to channels with radii $r < \bar{\rho}$ from the neighborhood Sdx of the point x in time dt, and it carries $3cv/(4\pi\bar{\rho}^3 m)\, dt\, SM_s \pi r^2 \varphi_s\, dr$ suspended particles. Over the same period of time, the same number of these channels becomes blocked up. In the Hagen–Poiseuille approximation [12], their carrying capacity at time t is equal to $\frac{3vc\pi}{32\bar{\rho}^3 m\mu}\, dt\, SM_s r^6 \varphi_s\, dr \frac{\partial p}{\partial x}$. Therefore, in time dt, the initial flux of the suspension $vSdt$ decreases by $\frac{3vc\pi}{32\bar{\rho}^3 m\mu}\, dt\, SM_s \int_0^{\bar{\rho}} r^6 \varphi_s\, dr \frac{\partial p}{\partial x}$. At the same time, in the case of a fixed pressure differential, Darcy's law leads to the equality

$$\frac{\partial v}{\partial t}\, dt = -\frac{1}{\mu}\frac{\partial k}{\partial t}\, dt \frac{\partial p}{\partial x}.$$

From this and from the above arguments, we conclude that

$$\frac{\partial k}{\partial t} = -v \frac{3\pi c}{32\bar{\rho}^3 m} M_s \int_0^{\bar{\rho}} r^6 \varphi_s\, dr. \tag{10}$$

In particular, it follows form (10) that the decrease in permeability is greater at the points where the flow velocity is higher and where the flow is more enriched in suspended particles. In media with low porosity, such variations become more abrupt.

System (1)–(10) is closed and, with appropriate boundary conditions and initial data, it completely determines the development of the sedimentation process. The "extra" condition (1) distinguishes physically meaningful solutions among the possible solutions of system (2)–(10). Extension to the multidimensional case is obtained by substituting $v = |\boldsymbol{v}|$ into the right sides of Eqs. (3)–(7) and (10), where $\boldsymbol{v}$ is the filtration velocity vector, by replacing the derivative with respect to x in (3) and on the left side of (8) by $\operatorname{div} \boldsymbol{v}$, and using the appropriate vectorial form of Darcy's law in (9).

Equilibrium Regimes. From a physical point of view, the equilibrium regimes are important because they describe the intrinsic properties of the process where the effects of temporal and spatial boundaries can be neglected. System (3)–(8) has a class of solutions in which the velocity of filtration v is a given constant and m, c, M, φ_s, and l depend on x and t only through the variable $\xi = x - \sigma t$, where σ is the needed velocity of a simple wave. In this case, Eqs. (3)–(7) reduces to

$$-\sigma m \frac{dc}{d\xi} + v \frac{dc}{d\xi} = -vc(1-c)\frac{q}{l}, \qquad \sigma \frac{dm}{d\xi} = vc\frac{q}{l},$$

$$\sigma \frac{d(M\varphi_s)}{d\xi} = vc \frac{3}{4\pi\bar{\rho}^3 l} r^2 \varphi_s H(\bar{\rho} - r) \Big/ \int_0^\infty r^2 \varphi_s\, dr, \qquad \sigma \frac{dM}{d\xi} = vc \frac{3}{4\pi\bar{\rho}^3}\frac{q}{l}, \tag{11}$$

$$-\sigma m \frac{dl}{d\xi} = vc\left(\frac{3l}{4\bar{\rho}^3} \int_0^{\bar{\rho}} r^4 \varphi_s\, dr \Big/ \int_0^\infty r^2 \varphi_s\, dr - q\right).$$

The first and second equations of system (11) yield the equality $m(1-c) + vc/\sigma \equiv a$, where a is a certain constant. To find a, we assume that the porous structure before the wave is in a certain perturbed state:

$$c = 0, \quad m = m_+, \quad M = M_+, \quad \varphi_s = \varphi_+(r), \quad l = l_+ \quad \text{as} \quad \xi = +\infty, \tag{12}$$

where $0 < m_+ < 1$ and $M_+ > 0$ are known parameters, $l_+ > 0$ is unknown, and $\varphi_+(r)$ is the known distribution of pores by radii. In this case, $a = m_+$, i.e., $m(1-c) + cv/\sigma = m_+$, and this leads to the following simple dependence of the concentration on the porosity and the velocity of wave propagation:

$$c = \frac{m_+ - m}{v/\sigma - m}. \tag{13}$$

Let a certain stable state appear past the wave:

$$c = c_+, \quad m = m_*, \quad M = M_*, \quad \varphi_s = \varphi_*(r) \quad \text{as} \quad \xi = -\infty. \tag{14}$$

Here $0 < c_+ < 1$ is a known constant, $0 < m_* < 1$ and $M_* > 0$ are unknowns, and $\varphi_*(r)$ is a unknown function. It follows from (13) that, in this case, the wave propagation velocity σ is given by

$$\frac{v}{\sigma} = \frac{m_+ - m_* + m_* c_+}{c_+}, \tag{15}$$

i.e., the flux of suspended particles vc_+ is divided into two components. The first of them $\sigma(m_+ - m_*)$ is entrained in the porous medium, and the second component $\sigma m_* c_+$ is carried by the fluid.

The second and fourth equations of system (11) yield the equality $(4/3)\pi\bar{\rho}^3 M - m \equiv \text{const}$. From this and from (12), we conclude that

$$M(\xi) = M_+ - \frac{3}{4\pi\bar{\rho}^3}(m_+ - m(\xi)). \tag{16}$$

The physical meaning of these relations is that the blocking of a pore is accompanied by a decrease in the pore space by the volume of the entrained particle.

Using (2) and integrating the third equation of system (11), we obtain

$$\ln(M\varphi_s) = A_1(r) + H(\bar{\rho} - r)\Big(-\frac{3v}{4\sigma\bar{\rho}^3} r^2 \int\limits_{\xi}^{\xi_0} \frac{c}{m}\,d\tau + A_2(r)\Big),$$

where $A_1(r)$ and $A_2(r)$ are arbitrary functions and ξ_0 is an arbitrary constant. Assuming that the integral converges at $\xi = +\infty$ and using condition (12), we obtain

$$\varphi_s = \frac{M_+\varphi_+(r)}{M(\xi)}\,(1 + H(\bar{\rho} - r)(\exp(-æ(\xi) r^2) - 1)). \tag{17}$$

The function $æ(\xi)$ included in this representation is defined by

$$æ(\xi) = \frac{3v}{4\bar{\rho}^3\sigma} \int\limits_{\xi}^{+\infty} \frac{m_+ - m}{v/\sigma - m}\,\frac{d\tau}{m}. \tag{18}$$

From the second equation of system (11), and also from (3), (17), and (18) it follows that

$$\frac{dm}{d\xi} = \frac{d}{d\xi}\Big(\frac{4}{3}\pi\bar{\rho}^3 M_+ \int\limits_0^{\bar{\rho}} \varphi_+(r)\exp(-æ(\xi)r^2)\,dr\Big).$$

Integrating this differential relation and taking into account that, according to (14), $c/m \to c_+/m_* > 0$ as $\xi \to +\infty$, we obtain the following representation for the porosity:

$$m(\xi) = m_* + \frac{4}{3}\pi\bar{\rho}^3 M_+ \int\limits_0^{\bar{\rho}} \varphi_+(r)\exp(-æ(\xi)r^2)\,dr. \tag{19}$$

We now consider the parameters m_*, M_*, and l_+ and the function $\varphi_*(r)$ which enter conditions (12) and (14) and representations (13) and (15)–(19). The quantity M_+ and the function $\varphi_+(r)$ specify the unperturbed state of the medium before the wave. To determine this state uniquely, it is necessary to specify one more characteristic, e.g., m_+. Then, the parameter l_+ must be such that equality (2) is satisfied at $\xi = +\infty$, i.e.,

$$m_+ = \pi l_+ M_+ \int_0^\infty r^2 \varphi_+(r)\, dr. \tag{20}$$

Then, in view of (3) and (11), we have

$$\frac{d}{d\xi}\Big(m - \pi l M \int_0^\infty r^2 \varphi_s\, dr\Big) = Q(\xi)\Big(m - \pi l M \int_0^\infty r^2 \varphi_s\, dr\Big)$$

with a certain function $Q(\xi)$. Using this differential relation, we conclude that equality (2) is a consequence of its particular form (20). Then, the function $l(\xi)$ can be obtained from (2) and (19).

Similarly, for the normalization condition (1) to be satisfied everywhere, it is sufficient to require that it be satisfied for a single value of the variable ξ, for example for $\xi = -\infty$. Then, it follows from (17), (18), and the properties of the function $æ(\xi)$ that as $\xi \to -\infty$, the limit of $M\varphi_s$ is equal to $M_*\varphi_*$, where

$$\varphi_*(r) = \frac{M_+}{M_*}\, \varphi_+(r) H(r - \bar{\rho}); \tag{21}$$

$$M_* = M_+ \int_{\bar{\rho}}^\infty \varphi_+(r)\, dr. \tag{22}$$

Passing to the limit $\xi \to -\infty$ in (16), we obtain the required representation for the parameter m_*:

$$m_* = m_+ - \frac{4}{3}\pi \bar{\rho}^3 (M_+ - M_*). \tag{23}$$

Thus, let the characteristics of the unperturbed porous structure $\varphi_+(r)$, M_+, and m_+, the particle size $\bar{\rho}$, and the concentration c_+ coming with the flow be given. We require satisfaction of the inequality

$$m_+ > \frac{4}{3}\pi \bar{\rho}^3 M_+ \int_0^{\bar{\rho}} \varphi_+(r)\, dr, \tag{24}$$

which means that the pore space of this structure contains the maximum possible pore volume to be blocked up by particles. (Otherwise, the equilibrium regime does not occur, and a rear front appears.) From (20)–(23) and (15), we successively obtain l_+, M_*, $\varphi_*(r)$, m_*, and σ. Then, it follows from condition (24) that $m_* > 0$. In this case, M_* and σ are positive. Relations (18) and (19) form a system of two nonlinear equations for $m(\xi)$ and $æ(\xi)$. Let a solution of this system exist, and $m(-\infty) = m_*$, $m(+\infty) = m_+$, $m_* < m(\xi) < m_+$, $æ(\xi) > 0$, $æ(-\infty) = +\infty$, and $æ(+\infty) = 0$. Then, (15) yields the inequality $v/\sigma - m(\xi) > v/\sigma - m_+ > 0$, and the function $c(\xi)$ determined from (13) satisfies the natural limitation on concentration $0 < c < c_+$ and the conditions $c(+\infty) = 0$ and $c(-\infty) = c_+$. It follows from (16), (23), (2), and (20) that $M(+\infty) = M_+$, $M(-\infty) = M_*$, and $l(+\infty) = l_+$. The two other limiting conditions $\varphi_s(+\infty, r) = \varphi_+(r)$ and $\varphi_s(-\infty, r) = \varphi_*(r)$ follow from (17) and (21).

The solution of system (18), (19) is constructed as follows. We introduce the parameter $s \in (0, +\infty)$ and the function

$$\Phi(s) = \int_0^{\bar{\rho}} \varphi_+(r) \exp(-sr^2)\, dr. \tag{25}$$

Let the functions $m(s)$, $\xi(s)$, and $æ(s)$ be determined by the rule

$$m(s) = m_* + \frac{4}{3}\pi\bar{\rho}^3 M_+ \Phi(s), \quad \xi(s) = \xi_0 + \frac{4\sigma}{3v}\bar{\rho}^3 \int_s^{s_0} m(y)\, \frac{v/\sigma - m(y)}{m_+ - m(y)}\, dy, \quad æ(s) = s, \tag{26}$$

where s_0 and ξ_0 are certain fixed numbers.

The function $m(s)$ decreases monotonically, $m(+\infty) = m_*$, and $m(+0) = m_+$ in view of (22) and (23). In this case, $v/\sigma - m(s) > 0$ and the integrand on the right side of (26) is positive for all $s > 0$. Therefore, $\xi(s)$ is defined for $s > 0$ and decreases monotonically. Using (22) and (23), we obtain

$$m_+ - m(s) = \frac{4}{3}\pi\bar{\rho}^3 M_+ \Psi(s), \qquad \Psi(s) = \int_0^{\bar{\rho}} \varphi_+(r)(1 - \exp(-sr^2))\,dr.$$

The function $\Psi(s)$, which is analytic in s, vanishes at $s = 0$, and $\Psi'(0) > 0$. Therefore, the representation $m_+ - m(s) = sA[1 + O(s)]$ with a positive constant A is valid in the neighborhood of the point $s = 0$. Hence it follows that $\xi(+0) = +\infty$ and, since $(v/\sigma - m_*)/(m_+ - m_*) = (1 - c_+)/c_+$ according to (15), we have $\xi(+\infty) = -\infty$. This implies that the inverse function $s(\xi)$ is defined on the axis $-\infty < \xi < +\infty$ and it decreases monotonically from $s(-\infty) = +\infty$ to $s(+\infty) = 0$. In this case, $m(s(\xi))$ increases monotonically from $m(s(-\infty)) = m_*$ to $m(s(+\infty)) = m_+$. Since, in the limit $s \to +0$, $\xi(s)$ behaves like $-A \ln s$, the integral of the function $m_+ - m(s(\xi))$ converges at $\xi = +\infty$. Next, by construction, $æ(s(\xi))$ satisfies the differential equation

$$\frac{dæ}{d\xi} = -\frac{3v}{4\bar{\rho}^3\sigma}\,\frac{m_+ - m(s(\xi))}{v/\sigma - m(s(\xi))}\,\frac{1}{m(s(\xi))},$$

and, therefore,

$$æ(s(\xi)) = æ_0 - \frac{3v}{4\bar{\rho}^3\sigma}\int_{\xi_0}^{\xi} \frac{m_+ - m(s(y))}{v/\sigma - m(s(y))}\,\frac{dy}{m(s(y))},$$

where $æ_0$ is a certain constant. Letting $\xi \to +\infty$ and taking into account the convergence of the integral and the fact that $æ(s(+\infty)) = 0$, we obtain the equality

$$æ_0 = \frac{3v}{4\bar{\rho}^3\sigma}\int_{\xi_0}^{+\infty} \frac{m_+ - m(s(y))}{v/\sigma - m(s(y))}\,\frac{dy}{m(s(y))},$$

i.e., $æ(s(\xi))$ satisfies the relation (18).

Thus, system (18), (19) is solvable and its solution has all the necessary properties. The nonuniqueness of the choice of ξ_0 in (26) is due to the fact that a simple wave admits a shift by an arbitrary constant. Representations (25) and (26) are convenient from a computational viewpoint because they admit expansions of the functions $m(s)$ and $\xi(s)$ in power series. As follows from (10), the equality $M_s = lM$, representation (17) for the function φ_s, and relations (26) between ξ and s, this parametrization leads to a simple expression for the porosity:

$$k(s) = k_+ - \frac{\pi}{8} M_+ \int_0^s l(y) \int_0^{\bar{\rho}} r^6 \varphi_+(r) \exp(-yr^2)\,dr\,dy. \tag{27}$$

Here k_+ is the permeability of the medium in its unperturbed initial state and the integral on the right side of (27) converges as $s \to +\infty$.

It can be shown that the free path of a particle of the suspension λ is defined by the formula $\lambda = l(1 - q)/q$, where q is given by (3), and it increases infinitely with passage of the wave. The behavior of the function $l(s)$ is more complicated, and here greatly depends on the contribution of small pores. Thus, if there is sufficient pore space for being potentially blocked up, i.e.,

$$l_+ \int_0^{\bar{\rho}} r^2 \varphi_s\,dr > \frac{4}{3}\pi\bar{\rho}^3 \int_0^{\bar{\rho}} \varphi_s\,dr,$$

then the inequality $l(-\infty) > l_+$ holds.

Conclusions. Let us point out several important properties of the sedimentation process. First of all, there is no distribution function whose form remains unchanged with time. A characteristic feature of pores of small radii is that their most representative part is "cut off" and the remaining part increases. Probably, this is a reason for the formation of distributions with several maxima in the sedimentation process.

Since the functions $m(s)$ and $k(s)$ are monotonic, representation (27) defines the monotonically increasing function $k(m)$. As the relations obtained above show, this dependence also includes information on the initial state of the pore structure. This, in particular, explains the diversity of the known empirical dependences $k(m)$.

As follows from (14), not only the porosity but also the concentration of suspended particles vary monotonically. It can be shown that plots of the functions $m(\xi)$, $c(\xi)$, and $k(\xi)$ have inflection points. Such points are also characteristic of experimental curves [1, 2]. The appearance of these points indicates that the flow has reached an equilibrium regime.

Integral models that are most often used to describe sedimentation processes include a number of additional empirical quantities apart from $k(m)$ [6–8]. The equilibrium regimes considered in the present paper are important because they can be used to obtain relations that express the kinetic parameters in terms of averaged flow characteristics [13].

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